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## 2010-02-21

# Abstraction: Einstein on Mathematics+Theory+Reality

Technorati Tags: **Albert Einstein**, **physics**, **formalized theories**, **interpretations of theories**, **reality**, **abstraction**, **models of theories**, **empiricism**

[**update 2010-02-23T19:46Z** I had to change the title of this post. My narration below revealed to me that *theory* is the statement (a claim) that connects the logical-forma *mathematics* to features of *reality*, and is therefore the statement of the theory that posits the validity of interpretation that connects to a model *in* reality (not *of* reality, so far). I'm also fixing a typo while I'm here.]

The abstraction of formal theories away from any appeal to nature is a theme for me. I claim that it gives us a powerful way to appreciate nature. I find it indispensible in teasing out what makes computers useful and what part software developers play in having that work for us. To my great pleasure, this is not a new or particularly radical notion at all, although relatively new to me. Albert Einstein had his own appreciation.

> "It is mathematics which affords the exact
> natural sciences a certain measure of security,
> to which without mathematics they could not attain."
> -- Albert Einstein, **1921, p.28**

In 1921, Albert Einstein, then 42, gave an address in which he explained what was necessary for him to appreciate about the difference between mathematics and theories about the physical universe in order to formulate the theory of relativity. He began by pointing out a peculiar situation around the applicability of mathematics to practical affairs.

> "An enigma presents itself which in all ages has agitated inquiring minds. How can it be that mathematics, being after all a product of human thought which is independent of experience, is so admirably appropriate to the objects of reality? Is human reason, then without experience, merely by taking thought, able to fathom the properties of real things?" [**1921**: p.28]

For Einstein, this is not a question; he will answer the second question in the negative. His analysis of the first question is foretold with this response:

> "As far of the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality." [**1921**: p.28]

He gives credit for clarity on the matter to the introduction of mathematical logic or "Axiomatics," put this way:

> "The progress achieved by axiomatics consists in its having neatly separated the logical-formal from its objective or intuitive content; according to axiomatics the logical-formal alone forms the subject-matter of mathematics, which is not concerned with

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**NuovoDoc: Design for Document System Interoperability**

**ODMA Interoperability Exchange**

**Orcmid's Lair**

**TROST: Open-System Trustworthiness**

> the intuitive or other content associated with the logical-formal." [**1921**: p.28]

At the time that Einstein speaks of this as such a powerful innovation, Whitehead and Russell's *Principia Mathematica* has only been in print since 1910. David Hilbert's famous 23 problems had just been announced in 1900 when Hilbert had already made a formal axiomatization of Geometry and demonstrated its completeness. In contrast, Kurt Gödel was about 15 when Einstein gave this talk. Alan Turing was 9. (My mother was 4.)

Einstein uses Geometry to illustrate the separation between the logical-formal mathematical expression of a theory in what he termed the modern formulation:

> "Geometry treats of entities which are denoted by the words straight line, point, etc. These entities do not take for granted any knowledge or intuition whatever, but they presuppose only the validity of the axioms … which are to be taken in a purely formal sense, i.e. as void of all content of intuition or experience. These axioms are free creations of the human mind. All other propositions of geometry are logical inferences from the axioms." [**1921**: p.30]

This completes the separation of mathematics, expressed as applications of mathematical logic, establishing the separation of mathematics from any reference to reality:

> "This view … purges mathematics of all extraneous elements, and thus dispels the mystic obscurity which formerly surrounded the principles of mathematics. But a presentation of its principles thus clarified makes it also evident that mathematics as such cannot predicate anything about perceptual objects or real objects. In axiomatic geometry, the words 'point,' 'straight line,' etc., stand only for empty conceptual schemata. That which give them substance is not relevant to mathematics." [**1921**: pp.30-31]

So how is the appropriateness of mathematics to real-world matters accomplished? Einstein sees it this way:

> "It is clear that the system of concepts of axiomatic geometry alone cannot make any assertions as to the relations of real objects of this kind [parts of the earth, measuring lines, etc.], which we will call practically-rigid bodies. To be able to make such assertions, geometry must be stripped of its merely logical-formal character by the co-ordination of real objects of experience with the empty conceptual frame-work of axiomatic geometry. To accomplish this, we need only add the proposition:—Solid bodies are related, with respect to their possible dispositions, as are bodies in Euclidean geometry of three dimensions. Then the propositions of Euclid contain affirmations as to the relations of practically-rigid bodies.
>
> "Geometry thus completed is evidently a natural science; we may in fact regard it as the most ancient branch of physics. Its affirmations rest essentially on induction from experience, but not on logical inferences only. We will call this completed geometry 'practical geometry,' and shall distinguish it in what follows from 'purely axiomatic geometry.' The question whether the practical geometry of the universe is Euclidean or not has a clear meaning, and **its answer can only be furnished by experience**." [**1921**: pp.31-32; emphasis mine]

Einstein prescribes a way, via necessarily-informal language, by which geometry can be interpreted to apply to aspects of the natural world with it asserted (the theory, in this matter) that the logico-formal conclusions of geometry are valid conclusions about the natural world. Verification of that

assertion depends on experience, not anything expressed in the formal geometry.

It is in this sense that I mean **Reality is the Model**. Einstein suggests that one can make a theory in physics by asserting this correspondence, an interpretation, between logico-formal geometry and the natural world. For Einstein, this practical geometry (or perhaps better, physical geometry) is a completion of logic-formal geometry. I prefer to have sharp separation with physical geometry as one distinct model of formal geometry. This makes room for yet other models not tied to natural objects and essentially independent of each other. (Consider the virtual geometry of worlds that exist only in images, such as the 3D experience of *Avatar*.)

However applied, the critical feature of the separation of mathematical (or mathematical-logico) formulations from interpretations in nature or elsewhere is the unexpected power it provides. I shall leave Einstein with the final word:

> "I attach special importance to the view of geometry which I have just set forth, because without it I should have been unable to formulate the theory of relativity." [**1921**: p.33]

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[**Einstein1921**]
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posted by orcmid at **2/21/2010 08:22:00 PM**


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